MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XXXV. *April* 9, 1875. No. 6.

PROFESSOR A. CAYLEY, Vice-President, in the Chair.

Carl Armbruster, Esq., F.C.S., Member of the Quekett Microscopical Club, 4 Grove Villas, The Grove, Hammersmith, W.,

Col. Archibald Campbell, Blythswood, and 2 Seamore Place, Curzon Street, W., and

David Winstanley, Esq., The Doctor's Cottage, Blackpool,

were balloted for and duly elected Fellows of the Society.

First Results of the Transit of Venus.

(*Extract from a Letter of M. D'Abbadie to the Astronomer Royal.*)

It may interest you to learn that M. Puiseux has given us the first French results for the Sun's parallax, using the observations of Pekin and St. Paul, all made with object-glasses of 216 millimetres. The parallax is 8″·879. With glasses of 162 millimetres, used by other observers in the same stations, the tenth is the same, and the hundredth less by one or two units. M. Fizeau is busy investigating the errors in the screws to be used in measuring the photographs. Our Committee has agreed, even before receiving your paper, to publish all observations in detail, leaving to others the task of combining and canvassing the final results.

Paris, 1875, *April* 13.

Observation of the Transit of Venus at Vizagapatam.

By A. V. Nursinga Row.

(*Abstract.*)

The observations were made with an equatoreal telescope, by Cooke, of 6 inches clear aperture, and about $7\frac{1}{2}$-feet focal length, and driven by clock-work.

The weather appeared to be favourable for the last thirty minutes of the transit, and the observations of the egress was successfully made. After the second external contact, when the limb of the Sun had resumed its natural appearance of an arc, a slight indentation was directly formed in the Sun's limb. This indentation was not so dense as that caused by the planet, but was more or less tending to an ash colour, and was apparently greater in arc than the previous one.

The following are the observed sidereal times :—

	h	m	s
Second internal contact	16	47	15·4
„ external „	17	15	27·2
Disappearance of the slight indentation noticed above	17	15	54·0

1875, *February* 20.

Account of Longitude Operations on the way from Mauritius homewards. By Lord Lindsay.

Chronometric Run—Mauritius to Aden.

At Mauritius.—Observatory at Belmont was connected with the railway station by a field telegraph, and railway lines; signals were exchanged with Belmont and the railway station, on evenings January 5 and 7, and the chronometers on board the " Dupleix," were compared with the standard chronometer immediately after the exchange of signals.

On the afternoon of January 1 the " Dupleix " sailed, and the following morning anchored off St. Denis. Here the thirteen chronometers of the Dutch Expedition were brought on board and compared; thus Belmont and St. Denis was fixed, and the thirteen Dutch chronometers were thrown into the expedition. The Dutch had good observations on the evening before.

On the evening of January 13 the " Dupleix " reached Seychelles, but on account of measles at Bourbon was put in quarantine. After very considerable difficulty on both sides, both from the local health authorities and the Captain of the " Dupleix," two chronometers were lowered astern, taken off by Captain Wharton of the " Shearwater, and compared with seven chronometers on board the " Shearwater." On return it was found that no sensible alteration had taken place in the rates of the chronometers during transport. Captain Wharton and his officers obtained equal altitudes of the Sun, the morning before and after; the following are the results (deduced from data just received from Captain Wharton) :—

Lord Lindsay's Standard Chronometer Fast of Local M.T. at Hondouls Jetty, Matre, at 10 P.M. *Jan.* 13, *from Obs. of Jan.* 12, 13, *and* 14, *by Officers of H.M.S. "Shearwater."*

From Obs. by	Chron. A	B	C	D	E	F	G
	m s	m s	m s	m s	m s	m s	m s
Capt. Wharton	7 39·43	7 39·34	7 39·29	7 39·39	7 39·21	7 39·08	7 39·43
Lieut. Langden	39·48	39·33	39·21	39·01	39·37	39·12	39 15
„ Wright	38·92	38·82	38·76	38·91	38·71	38·57	38·96
„ Moore	39·30	39·18	39·02	39·28	39·09	38·96	39·33
Mean	7 39·28	7 39·16	7 39·09	7 39·18	7 39·09	7 39·93	7 39·26

These results are entirely satisfactory, and showing that little has been lost from our not being allowed to land an instrument at Seychelles.

On the evening of the 20th the "Dupleix" arrived at Aden, and within three hours good observations and comparisons were made.

Dr. Low transhipped and continued his voyage to Suez.

The weather was very cloudy at Aden. Observations were obtained on the evenings of 25th, 28th, 29th (early morning of 30th), 30th (early morning of 31st), and 31st (early morning of February 1). No other opportunities for observations occurred.

The whole of the chronometers were compared every day, and immediately before and after each set of telegraph exchanges.

Telegraph exchanges were made with Suez on Saturday, January 30 (early Sunday morning), on January 31 (Sunday evening), and January 31 (early Sunday morning), and with Bombay on January 31, immediately before the exchanges with Suez, both on Sunday evening and Monday morning.

The exchanges with Suez were found to be very satisfactory. The results for "Current Time" turn out as follow:—

	m	s
From exchanges of Sunday morning	+ 0	47
„ „ Sunday evening	+ 0	42
„ „ Monday evening	+ 0	45

On February 1, I left for Suez. On arrival, February 8, found a steamer of the Khedive's, and an officer of Engineers, to convey my baggage ashore. Obtained observations same evening.

On February 9, Dr. Low and I had two long series of observations for personal equation in observing stars.

On the 10th an artificial cable was prepared, and four series of determinations made for personal equation in observing Thomson's galvanometer signals.

On the 11th Dr. Low was laid up unwell and unable to observe.

I had the altazimuth packed up and conveyed to Mr. Hunter's station and mounted there, and at night made a series of time observations, as follows:—

(1). At Hunter's Station.
Compared 9 chronometers.
Made series of time observations.
Compared 9 chronometers.
Conveyed 9 chronometers to Gill's station.

(2). At Gill's Station.
Compared 9 chronometers.
Observed double series for time determination with Low's Transit.
Compared 9 chronometers.
Conveyed 9 chronometers to Hunter's Station.

(3). At Hunter's Station.
Compared 9 chronometers.
Made a series for time determination.
Compared 9 chronometers.

This occupied me all night long.

Next day took a truck, had all packed, and on 13th started for Cairo, arrived there late that night.

February 14. Got carts, &c., and conveyed chronometers and apparatus to Observatory. Saw General Stone, who asked me to look at and report upon their instruments.

February 15. Spent all day at Observatory, examined and reported on it and instruments.

February 16. Left for Alexandria.

February 17 and 18. Installing myself here, building pillar, and waiting a clear night.

February 19. Exchanged signals with Suez: very fine observations before and after.

February 20. Exchanged signals with Suez: very fine observations before and after.

February 21 and 22. Relays end for end, and night of 22nd cloudy.

February 23. A splendid set of exchanges, air perfectly still, and fine observations both before and after.

February 24. Exchanged signals, but no observations were possible here from cloud.

February 25. Good observations here and at Suez, but owing to a fault on the line no exchanges possible—resolved to adopt Low's observations of 24th, and to interpolate the rates of the 9 chronometers I have here from observations of 23rd and 25th for error of chronometer on 24th. Giving this latter a low weight, the splendid night of 23rd will equalise the weights of the first two with the last two nights.

February 26. Low knocked up and unable to work.

February 27. Low packed up. Same night observed fine

series for time in order to exchange with Berlin; but owing to a fault, afterwards found to be at Malta, no signals could be exchanged.

February 28. A splendid set of observations and exchanges with Berlin, the observations and exchanges lasting all night long.

March 1 and 2. Hopelessly cloudy, reduced the exchanges between Suez and Alexandria.

The results for Current Time are—

		Difference from Mean.
	s	s
From exchanges of 19th	+ 0·057	+ 0·027
„ „ 20th	+ 0·022	− 0·008
„ „ 23rd	− 0·011	− 0·048
„ „ 24th	+ 0·053	+ 0·023
Current Time	0·030	6

The probable error of the result will be very small, especially when we consider that no chronograph is used.

March 3. Low and I made two series of comparisons of personal equation; and between the series I made a series of observations with his transit to find if there was any difference in my personal equation in using the altazimuth and the broken transit.

(This was necessary in the connection of the two stations, Gill's and Hunter's, at Suez).

March 4. Low left for Malta, I for Alexandria.

March 5. Selected site for base line with General Stone.

March 6. Returned to Alexandria, exchanged signals with Berlin. Storm of wind and rain prevented observations.

Photography in the Transit of Venus. By R. A. Proctor.

I do not know whether Capt. Abney's remarks (p. 309) relate to a paper by the Chief of the Washington Observatory, which appeared in *Harper's Weekly Magazine*, or to an essay of mine quoting Prof. Newcomb's opinion in one of the quarterly magazines. Possibly to neither, as the great difficulty dwelt on by Newcomb is "placed on one side" in Capt. Abney's paper. It may be desirable to note the nature of that difficulty. The American Transit Committee, after many experiments and long inquiry, came to the conclusion that the diameter of the Sun, as depicted by the photoheliograph, could not be ascertained (at least with the extreme accuracy necessary for determining the solar parallax) either by calculation (depending on the optical

adjustments) or by direct measurement, or by any practical contrivance, such as photographing a scale. On the other hand, they consider that reliance can be placed on the calculated scale of a picture taken at the principal focus, while the centres of the disks of *Venus* and the Sun can be determined accurately, because each centering results, not from a single pair of measures, but from as many all round each limb as the observer may wish to make. Of course the position of the centres may be determined in the same way, from a photoheliographic picture; but no advantage results if there is no trustworthy scale of measurement. This, then, is the point at issue, viz., whether a trustworthy scale can be obtained. The available methods are (i) calculation, (ii) measurement of the pictured solar diameter, and (iii) photographing a scale. As to the first, I apprehend that there can be no comparison in point of exactness between the calculated scale of a picture at a principal focus, and that of a picture optically enlarged: it is only necessary to consider the optical adjustments and relations involved, to be assured of this. As to the second method, it matters little whether photographic irradiation be large or small; for at the lowest estimate ever yet made, the effects of irradiation must be fatal in such a problem as determining the solar parallax: apart from this, we now have evidence showing that the photographic Sun is really larger than the optical Sun. As to the third method, it seems sufficient to note that the use of a photographed scale introduces of itself a probable error as large as that in single measurements of the photographed disk of the Sun or *Venus*; and such an error would be fatal in a problem of this kind.

The fact pointed out by Capt. Abney that daguerreotypes "are subject to much greater fluctuations of expansion than are glass negatives," seems to counterbalance the superiority claimed for them (by Sir G. Airy, I think, at the January meeting) in point of definition.

It may be hoped that before the Transit of 1882 photographers and astronomers will have decided on some one method of photographing *Venus* in transit. The qualities of the various methods employed on the present occasion will be sufficiently indicated during the examination of the complete series of transit observations. One point seems already clear, viz., that contacts determined from photographic records differ from contacts observed with the telescope, the photographic Sun being larger than the Sun we see. Hence greater reliance will probably be placed on mid-transit photographs. This country is afforded a noble opportunity of serving science by providing Southern stations for this purpose in 1882; an opportunity of which it may be hoped that she will avail herself.

Worthing, 1875, *April* 3.

On the Relative Rectilinear Motion of the Components of 61 Cygni.

By J. M. Wilson, Esq., M.A.

I have lately examined into the relative motion of the components of 61 *Cygni*, with the intention of ascertaining how far recent measures confirm Struve's conclusion that this motion is rectilinear. The very great common proper motion of these stars, that of one being given by Struve as $517'' \pm 10''$ per century in the direction $51° 16' \pm 1°$, and that of the other as $509'' \pm 10''$ in the direction $53° 38' \pm 1°$, gives an enormous probability to the hypothesis that these stars are physically connected. Struve computed it arithmetically, and concludes with the words, "nexus itaque physicus inter stellas 61 *Cygni* plus centies millies est probabilior, quam crastinus Solis ortus secundum experientiam historicam plus quinque millium annorum exspectandus."

But on examining into the observations on the relative distance and position of these stars from the year 1753 to 1851, he found that they were adequately represented by motion in a *straight line*. His words are, "Consensus, qui inter relationes observatas et eas, quæ calculo deductæ sunt, obtinet, talis est, ut nullus dubito quin declarem, motum relativum duarum stellarum in 61 *Cygni* nexarum ne minimam quidem a linea recta deflexionem quæ in observationibus cognosci posset, centum annorum spatio prodidisse. Res sane maximopere mirabilis mihique valde inexspectata."

In the following investigation, have made use of all the measures accessible to me, most of which were kindly sent by my friend Mr. Gledhill. They include a large number of observations since 1851, and many also before 1851 which Struve did not use; as he appears to have employed, for the years 1828 to 1851 only his own observations. In using these measures there is no certain method of deciding what weights ought to be given to the measures. I have, however, given great weight to Struve's measures from 1819 to 1836, then Dawes and Mädler become the principal authorities till 1854, and latterly Dembowski who measured this star with very great care on many nights, and at the present time Gledhill and Main.

The position and distance in 1753, &c., are to be calculated from Bradley's observations, that

$$\Delta\alpha = 14''{\cdot}4 \pm 1'' \text{ and } \Delta\delta = 15''{\cdot}7 \pm 1''.$$

The former of these is the mean of only two observations, $15''$, and $13''$ 8, and the latter is a single observation. These give an angle of $36° 0' \pm 3° 40'$, and the distance at the same time is $19''{\cdot}40$, with a considerable probable error.

The annexed table gives all the measures with which I am acquainted, and for comparison the places computed on the hypothesis of the uniform relative rectilinear motion along a line

whose equation (referred to axes passing through the principal star, the axis of x being the declination circle of the epoch 1870, which is the zero of angle, and that of y in the direction 90°) is

$$y = x \tan 166° 58' + 15{\cdot}60.$$

The places are calculated for A.D. τ from the formula

$$\tan \theta = \frac{y}{x} = \frac{12{\cdot}1664 + {\cdot}04292\ (\tau - 1750)}{14{\cdot}8336 - {\cdot}18542\ (\tau - 1750)}$$

$$r = y \operatorname{cosec} \theta.$$

It will be observed that the effect of precession has to be taken into account, and that a correction has to be applied to all the observations of position to bring them up to the epoch 1870.

Date.	$\theta°$ Obs.	$\theta°$ Corr.	$\theta_1°$ Calc.	$\theta_1 - \theta°$	r'' Obs.	r_1'' Calc.	$r_1'' - r''$	
1753·8	36·00 ±	35·42 ±	41·11	+5·69	19·41 ±	18·75	− ·66	Bradley
78·7	51·0 ±	50·55 ±	54·62	+4·07	15·27 ±	16·43	+1·16	Mayer
81·9	53·81	53·37	56·62	+3·25	16·33	16·21	− ·12	Herschel
1802·3	74·36 ±	74·02 ±	70·38	−3·64	16·8 ±	15·30	−1·5	Piazzi
06·3	67·54 ±	67·22 ±	73·23	+6·01	14·85 ±	15·63	+ ·78	„
12·3	79·11	78·82	77·52	− ·96	16·74	15·20	−1·54	Bessel
19·92	82·96	82·72	82·98	+ ·26	15·20	15·28	+ ·08	Σ
21·62	84·38	84·14	84·18	+ ·04	15·02	15·32	+ ·30	Σ
22·9	84·68	84·44	85·08	+ ·64	15·42	15·36	− ·06	H & S
25·7	86·93	86·71	87·04	+ ·33	15·44	15·43	− ·01	S
28·72	89·40	89·19	89·20	+ ·01	15·31	15·55	+ ·24	Σ
30·64	90·33	90·13	90·43	+ ·30	15·70	15·63	− ·07	Dawes
·84	90·35	90·15	90·57	+ ·42	15·63	15·64	+ ·01	Σ
31·74	91·16	90·97	91·18	+ ·25	15·63	15·68	+ ·05	Σ
·74	90·70	90·51	91·18	+ ·67	15·45	15·68	+ ·23	H
32·57	90·91	90·72	91·75	+1·03	...	...	...	H
·77	92·05	91·86	91·88	+ ·02	15·79	15·73	− ·06	Σ
33·80	92·78	92·60	92·57	− ·03	15·88	15·78	− ·10	Dawes
35·65	93·83	93·66	93·79	+ ·13	15·96	15·88	− ·08	Σ
36·57	94·41	94·24	94·38	+ ·14	16·08	15·93	− ·15	Σ
37·63	95·15	94·99	95·07	+ ·08	16·27	15·97	− ·30	Encke
·71	95·45	95·29	95·12	− ·17	15·91	15·97	+ ·06	Galle
38·38	95·32	95·16	95·55	+ ·39	16·20	16·03	− ·17	Bessel
·73	96·06	95·90	95·78	− ·12	16·76	16·05	− ·71	Galle

Date.	θ° Obs.	θ° Corr.	θ_1° Calc.	$\theta_1 - \theta^\circ$	r'' Obs.	r_1'' Calc.	$r_1'' - r''$	
1840·05	97·10	96·95	96·63	− ·32	16·01	16·14	+ ·13	Kaiser
·73	97·25	97·10	97·06	− ·04	16·4	16·19	− ·21	Dawes
41·49	98·53	98·38	97·55	− .83	16·49	16·24	− ·25	Mädler
·87	97·94	97·79	97·78	− ·01	16·55	16·26	− ·29	Dawes
42·62	99·03	98·89	98·24	− ·65	16·86	16·33	− ·53	Mädler
43·64	99·51	99·38	98·89	− ·49	16·5	16·39	− ·1	„
·76	98·90	98·77	98·96	+ ·19	16·78	16·39	− ·39	Dawes
·98	99·69	99·56	99·11	− ·45	...	...	...	„
44·34	99·42	99·29	99·32	+ ·03	16·68	16·44	− ·24	O Σ
·48	100·10	99·97	99·40	− ·57	16·35	16·45	+ ·10	Mädler
45·06	99·86	99.74	99·75	+ ·01	16·36	16·48	+ ·12	„
46·07	100·97	100·85	100·37	+ ·48	17·07	16·56	− ·51	Dawes
48·07	99·8	99·69	101·56	+1·96	16·4	16·70	+ ·3	Smyth
·30	101·95	101·84	101·70	− ·14	17·0	16·73	− ·27	Dawes
·49	101·47	101·36	101·81	+ ·45	17·11	16·74	− ·37	O Σ
50·88	103·87	103·77	103·19	− ·58	17·04	16·93	− ·11	Dawes
·94	103·06	102·96	103·24	+ ·28	16·79	16·94	+ ·17	Mädler
51·81	103·67	103·58	103·74	+ ·16	17·32	17·03	− ·29	O Σ
·90	103·66	103·57	103·79	+ ·21	16·90	17·03	+ ·13	Mädler
52·68	104·69	104·60	104·23	− ·37	17·11	17·10	− ·01	Mädler
·76	104·30	104·23	104·27	+ ·04	17·40	17·11	− ·29	Jacob
53·13	104·45	104·36	104·48	+ ·12	16·90	17·13	+ ·23	Mädler
·26	104·39	104·30	100·55	+ ·25	17·24	17·14	− ·10	Dawes
·80	103·70	103·62	104·87	+1·25	17·0	17·20	+ ·2	Smyth
·89	104·73	104·65	104·92	+ ·27	17·68	17·20	− ·48	Jacob
·94	104·85	104·77	104·95	+ ·18	17·28	17·21	− ·07	Mädler
54·55	105·02	104·94	105·28	+ ·34	17·63	17·26	− ·37	Mädler
·73	105·58	105·50	105·38	− ·12	17·29	17·28	− ·01	Dembowski
·98	105·69	105·61	105·52	− ·09	17·29	17·30	+ ·01	„
55·55	105·60	105·53	105·87	+ ·34	17·50	17·36	− ·14	Secchi
·99	105·9	105·83	106·09	+ ·26	17·94	17·40	− ·54	„
56·63	105·2	105·13	106·44	+1·31	17·89	17·46	− ·43	„
57·59	107·86	107·80	106·95	− ·85	17·62	17·55	+ ·07	Dembowski
62·97	109·50	109·47	109·76	+ ·29	18·36	18·08	− ·28	„
65·15	110·64	110·62	110·85	+ ·23	18·55	18·31	− ·24	„
65·75	109·7	109·68	111·15	+1·47	...	...	...	Talmage
·76	111·3	111·28	111·15	− ·13	18·75	18·36	− ·37	„
66·73	112·8	112·78	111·62	−1·16	18·81	18·49	− ·32	„

Date.	$\theta°$ Obs.	$\theta°$ Corr.	$\theta_1°$ Calc.	$\theta_1-\theta°$	r'' Obs.	r_1' Calc.	$r_1''-r''$	
1866·84	111·8	111·78	111·63	− ·15	18·84	18·49	− ·35	Secchi
67·16	111·78	111·77	111·83	+ ·06	18·73	18·52	− ·21	Dembowski
68·60	112·32	112·32	112·51	+ ·20	18·70	18·70	− ·00	Main
69·28	113·27	113·27	112·83	− ·44	18·91	18·76	− ·15	Dembowski
·58	112·31	112·31	112·97	+ ·66	18·65	18·79	+ ·14	Main
70·49	112·37	112·37	113·40	+ 1·03	19·27	18·90	− ·37	„
·53	113·9	113·9	113·41	− ·49	19·15	18·90	− ·25	Gledhill
71·50	114·08	114·09	113·87	− ·22	19·27	19·02	− ·25	„
72·72	113·6	113·61	114·43	+ ·82	18·93	19·15	+ ·18	Talmage
·73	114·45	114·46	114·43	− ·02	19·00	19·15	+ ·15	Wilson
·78	114·15	114·16	114·44	+ ·28	19·77	19·16	− ·61	Main
73·05	114·59	114·60	114·55	− ·05	19·38	19·19	− ·19	Dembowski
·75	114·97	114·99	114·88	− ·11	19·36	19·26	− ·10	Gledhill
74·92	115·4	115·42	115·43	+ ·01	19·52	19·41	− ·11	„

It must be observed that in the above calculation the observations of angles were allowed more weight than those of distances, it being difficult, as is shown by the discordant observations, to determine the distance of a 19″ star within 0″·1, or even 0″·2. And further, no weight was attached to early observations previous to 1819. The methods adopted were partly graphical, partly analytical. The conclusions, therefore, that seem to follow from the table, are as follow: the hypothesis of uniform rectilinear motion satisfies the observation of *angle* during the present century with sufficient accuracy, but it makes the *distances* a little less than the observed distances during the last 25 years. But this hypothesis fails to satisfy the observations of the last century with fair accuracy, the errors increasing in earlier observations, and Struve's hypothesis fails equally to satisfy present observations.

The result obtained by Struve from a discussion of the most probable result of the early measures combined with his own was, that the motion was in a straight line, which had a minimum distance of 14″·74 in 1805·35 in the direction 71° 28′, and my line has a minimum distance of 15″·19 in 1811·58 in the direction of 76° 58′. The combination of these would indicate that the motion has deviated slightly from rectilinearity, the earlier described portions of the path being inclined to the later portions at an angle of about 5° 30′, and making the whole path concave to the principal star. But this conclusion rests almost entirely on the first three observations of the list, and must, therefore, be accepted with some hesitation.

What is certain is, that the motion deviates extremely little from rectilinearity, considering that so large an angle has now

been described. The star ought to be measured repeatedly with great care, and with the best instruments, for some years to come, and then the question of its motion may be advantageously re-discussed.

A diagram is annexed to show the path that has been at present described.

Temple Observatory, Rugby,
1875, *March.*

Observations of the Satellites of Saturn made in 1874 *at the Washington Observatory.*

(*Communicated by the Director of the Observatory.*)

In the column "No." the figure denotes either the number of independent measures of position-angle, or the number of measures of double distance. In the column "wt." the number denotes the goodness of the images on a scale from 5 = perfect to 1 = very bad. In the column "Obser." N denotes Newcomb and H Holden.

The value of 1 revolution of the micrometer-screw is $9''{\cdot}9482 \pm 0''{\cdot}0049$.

Mimas.

1874.	Wash. M.T. h m	Pos. Ang. °	No.	Wt.	Wash. M.T. h m	Pos. Ang. °	No.	Wt.	Obser.
Sept. 2	9 53	274·7	2	2	10 12	2·856	2	2	N
26	10 12	92·9	4	2	10 1	2·700	3	2	„
Oct. 15	7 8	94·8	2	2	7 17	2·714	1	2	„

Enceladus.

Aug. 29	9 56	268·3	3	...	10 35	3·346	2	...	N
31	10 23	85·1	3	...	10 31	2·982	2	...	„
Sept. 2	9 58	252·4	2	...	10 5	2·304	2	...	„
22	8 54	86·6	3	...	9 30	3·209	2	...	„
23	7 59	291·0	3	...	8 32	2·640	2	...	„
24	9 16	260·3	3	...	10 4	2·981	2	...	„
25	7 56	105·5	3	...	8 54	2·980	2	...	„
26	8 4	55·4	3	3	8 44	1·825	2	3	„
Oct. 1	8 25	272·8	3	...	9 23	3·502	1	...	„
3	7 23	82·5	2	...	8 30	3·142	2	...	„
14	7 37	89·3	2	...	8 22	3·193	2	...	„
15	7 26	300·2	2	...	7 48	2·087	2	...	„
17	8 12	115·7	2	1	...	...	...	...	H

Tethys.

Aug. 29	9 50	299·0	2	...	10 27	2·561	2	...	N
31	9 28	320·1	3	...	10 8	1·663	2	...	„
Sept. 2	9 13	13·5	2	...	9 47	1·377	2	...	„
3	8 42	213·8	2	...	9 19	1·639	2	...	„
22	8 33	221·6	3	...	9 8	2·773	2	...	„
23	8 19	77·9	3	...	9 0	3·230	3	...	„
24	9 4	266·6	3	...	9 43	3·959	2	...	„
25	7 59	87·4	2	...	9 0	4·012	2	...	„
26	8 35	272·7	3	...	9 41	4·897	2	...	„
Oct. 1	8 19	109·8	3	...	9 17	3·330	2	...	„
3	7 17	116·4	2	...	8 23	2·602	2	...	„
13	7 38	270·5	3	...	8 14	4·100	2	...	„
14	7 34	93·9	2	...	8 6	4·212	2	...	„
15	7 30	277·4	2	...	7 54	4·257	2	...	„
17	7 18	283·8	4	...	8 15	3·860	3	...	H
20	6 38	114·3	4	...	7 6	3·019	3	...	H
22	6 49	134·2	3	1	...	...	...	...	H
23	6 46	330·9	5	...	6 56	1·446	3	...	H

Dione.

1874.	Wash. M.T. h m	Pos. Ang. °	No.	Wt.	Wash. M.T. h m	Pos. Ang. °	No.	Wt.	Obser.
Aug. 29	10 3	107·0	3	...	10 42	5·024	2	...	N
31	9 35	56·8	3	...	10 14	2·779	2	...	„
Sept. 2	9 8	280·1	3	...	9 41	5·697	2	...	„
3	8 30	82·2	2	...	9 0	4·590	2	...	„
22	8 39	38·0	3	...	9 15	3·350	2	...	„
23	8 6	135·5	3	...	8 40	2·488	2	...	„
24	8 57	284·7	3	...	9 32	5·040	2	...	„
25	8 3	87·4	2	...	9 4	5·164	2	...	„
26	8 20	226·1	3	...	9 21	3·060	2	...	„
Oct. 1	8 13	112·6	3	...	9 10	4·093	2	...	„
3	7 12	66·0	2	...	8 17	3·436	2	...	„
13	7 39	276·0	3	...	...	...	...	...	H
...	...	...	...	...	8 19	5·462	2	...	N
14	7 31	74·1	2	...	8 0	3·659	2	...	N
15	7 34	156·1	2	...	8 0	1·891	2	...	„
17	7 8	90·0	4	...	7 47	5·255	3	...	H
20	6 45	101·1	3	...	7 26	5·269	4	...	H
22	6 38	19·7	5	1	...	...	...	...	H
23	7 21	117·8	3	4	...	...	...	...	H
...	...	...	...	...	7 43	3·612	2	...	N

Rhea.

Aug. 29	9 45	287·6	3	...	10 19	6·705	2	...	N
31	9 22	99·96	3	...	9 59	7·852	2	...	„
Sept. 2	9 4	273·36	3	...	9 34	8·039	2	...	„
3	8 36	312·73	2	...	9 9	3·736	2	...	„
22	8 48	83·3	3	...	9 23	6·562	2	...	„
23	8 12	112·1	3	...	8 48	6·090	2	...	„
24	9 9	251·5	3	...	9 52	4·987	2	...	„
25	7 51	282·8	3	...	8 48	7·390	2	...	„
26	8 15	34·7	3	...	9 16	2·953	2	...	„
Oct. 1	8 7	80·8	3	...	9 2	6·212	2	...	„
3	7 9	241·6	2	...	8 10	4·300	2	...	„
13	7 42	280·8	3	...	8 25	7·411	2	...	„
14	7 27	11·4	3	...	7 54	2·424	2	...	„
15	7 37	94·4	2	...	8 15	7·578	2	...	„
17	7 25	267·1	2	...	...	...	...	...	H
20	6 52	106·8	3	...	7 44	6·233	3	...	H

1874.	Wash. M.T. h m	Pos. Ang. °	No.	Wt.	Wash. M.T. h m	Pos. Ang. °	No.	Wt.	Obser.
Oct. 22	6 57	278·7	3	4	...	...	...	...	H
23	6 38	354·4	3	...	7 11	2·342	3	...	H
				Titan.					
Aug. 29	10 9	185·4	3	...	10 51	5·966	2	...	N
31	9 15	238·80	3	...	9 46	13·650	3	...	„
Sept. 1	9 24	268·96	3	...	9 35	17·193	2	...	„
2	8 55	276·30	3	...	9 26	18·414	2	...	„
3	8 24	284·06	3	...	8 48	17·009	2	...	„
22	9 1	25·8	3	...	9 37	5·945	2	...	„
23	8 24	67·1	3	...	9 11	10·218	3	...	„
24	9 25	83·3	3	...	10 13	15·050	3	...	„
25	8 22	91·7	3	...	8 39	17·829	3	...	„
26	8 29	98·7	3	...	9 32	18·494	3	...	„
Oct. 1	8 0	240·6	3	...	8 54	9·113	2	...	„
2	8 1	259·5	2	...	8 20	13·556	3	...	„
3	7 3	269·5	3	...	7 45	16·720	3	...	„
13	7 51	107·3	3	...	8 32	16·254	2	...	„
14	7 44	116·9	3	...	8 13	12·560	3	...	„
15	7 41	138·0	3	...	8 7	7·985	3	...	„
17	7 31	241·5	3	...	...	...	...	...	H
19	8 37	270·53	3	...	8 44	16·414	3	...	N
20	8 37	277·96	3	...	8 25	14·638	5	...	„
23	7 27	323·72	3	...	...	...	...	...	H
...	...	...	...	...	7 50	10·436	3	...	N
27	7 35	92·22	3	...	7 42	17·031	6	...	„
				Hyperion.					
July 14	13 12	132·2	2	...	13 2	9·940	1	...	N
The date is possibly July 13.									
Oct. 2	8 32	93·7	3	...	8 36	23·788	3	...	„
3	7 28	98·0	2	...	8 5	23·339	1	...	„
				Iapetus.					
Sept. 26	8 9	54·1	3	3	8 53	7·897	3	3	N
Oct. 1	7 48	84·0	2	...	8 42	26·424	2	...	„
2	7 56	85·2	2	...	8 7	29·961	4	...	„
3	7 38	86·5	3	...	7 56	33·135	5	...	„

1874.	Wash. M.T. h m	Pos. Ang. °	No.	Wt.	Wash. M.T. h m	Pos. Ang. °	No.	Wt.	Obser.
Oct. 13	8 3	92·88	2	...	...	...	...	...	N
14	8 40	93·28	2	...	8 30	50·326	5	...	„
15	8 31	93·84	2	...	8 52	50·137	6	...	„
19	8 30	95·84	2	...	8 15	45·536	5	...	„
20	8 6	96·32	2	...	8 2	43·602	5	...	„
23	8 3	98·37	2	...	8 16	36·387	5	...	„
27	7 16	103·31	4	...	7 28	24·168	5	...	„

On an Investigation of Bessel's on the Refraction through an Atmosphere. By E. Neison.

In the *Astronomische Nachrichten*, No. 263, is an investigation of Bessel's on the limits of a possible lunar atmosphere, in which he determines the greatest possible density any atmosphere to the Moon can possess, and his conclusions have been widely accepted and quoted, though generally without any indications of how they were obtained, or where they appeared. Having lately become aware of the complete nature of Bessel's labours, their discordance with those given in the *Monthly Notices*, vol. xxxiii. p. 10, demanded attention. On examining Bessel's investigation, it was found that he had assumed the condition of a uniform temperature, as being the most favourable to a great surface density for a given horizontal refraction, whereas it is the most unfavourable; whilst it appeared certain he had, in quoting an equation from Laplace (*Méc. Cel.* iv. p. 252) employed one constituent without modifying it in the manner necessary when applied as he proceeded to do. As the uncertainty arising from such discrepancies is very inconvenient, an independent derivation of the true result has been made, omitting the constituent in question.

Assume

a = radius of Moon (or planet);

g, δ, p, t, μ = force of gravity, density, pressure, temperature, and index of refraction, at a height x above the surface;

g_o, δ_o, p_o, t_o, μ_o = the same, at the surface.

Then the equation

$$dp = -g.\delta.dx,$$

changing the variable to u, where $u = \frac{1}{a+x}$ and $dx = -\frac{du}{u^2}$, and putting for g its equivalent $a^2\, u^2\, g_o$, becomes

$$dp = g_o a^2 \delta \, du. \tag{1}$$

Dividing by the value of p from the equation

$$\frac{p}{p_o} = \frac{\delta}{\delta_o} \cdot \frac{1+\epsilon t}{1+\epsilon t_o}, \tag{2}$$

where ϵ is the rate of expansion of gases for one degree centigrade, the equation becomes

$$\frac{dp}{p} = g_o \frac{\delta_o}{p_o} a^2 \frac{1+\epsilon t_o}{1+\epsilon t} \, du; \tag{3}$$

and by integration and determining the constant from the condition that ϵ equals $\log p$ when $u = \frac{1}{a}$, we find

$$\frac{p}{p_o} = e^{g_o \frac{\delta_o}{p_o} (1+\epsilon t_o) \int \frac{du}{1+\epsilon t}}$$

where the inferior limit of the integral is $\frac{1}{a}$.

Hence

$$\frac{\delta}{\delta_o} = \frac{1+\epsilon t_o}{1+\epsilon t} e^{\frac{\delta_o}{p_o} (1+\epsilon t_o) \int \frac{du}{1+\epsilon t}}, \tag{4}$$

and then assuming the further condition that the temperature is constant, so that $t = t_o$, and restoring to u its value, we obtain

$$\frac{\delta}{\delta_o} = e^{-g_o \frac{\delta_o}{p_o} \cdot \frac{a}{a+x} \cdot x}; \tag{5}$$

or assuming $1 - s = \frac{a}{a+x}$, and denoting the constant quantity $g_o \frac{\delta_o}{p_o}$ by β,

$$\delta = \delta_o \, e^{-\beta s}. \tag{6}$$

From the theory of the refraction of light through a spherical atmospheric envelope, if ζ be the refraction, i the angle of incidence, and z the zenith distance,

$$d\zeta = -\tan i \, \frac{d\mu}{\mu},$$

and

$$\mu (a+x) \sin i = \mu_o \cdot a \sin z;$$

whence, substituting in the former the value of tan i derived from the latter, and writing for $\frac{a}{a+x}$ the value $1-s$, after reduction the equation becomes

$$d\zeta = -\frac{(1-s)\sin z\,\frac{d\mu}{\mu}}{\left\{\cos^2 z - \left(1-\frac{\mu^2}{\mu_o{}^2}\right) + (2s-s^2)\sin^2 z\right\}^{\frac{1}{2}}}. \qquad (8)$$

From the law of the dependence of the refraction upon the density,

$$\mu^2 - 1 = \frac{2a}{1-2a}\cdot\frac{\delta}{\delta_o};$$

by differentiating, and (as a is supposed very small) replacing $2a\left(1-\frac{\delta}{\delta_o}\right)$ by a,

$$\frac{d\mu}{\mu} = \frac{a}{1-a}\cdot\frac{d\delta}{\delta_o},$$

and then, by substituting in equation (8),

$$d\zeta = -\frac{a}{1-a}\cdot\frac{(1-s)\sin z\,\frac{d\delta}{\delta_o}}{\left\{\cos^2 z - 2a\left(1-\frac{\delta}{\delta_o}\right) + (2s-s^2)\sin^2 z\right\}^{\frac{1}{2}}}, \qquad (9)$$

the differential equation to the refraction obtained by Laplace, (*Méc. Celeste*, Book x., c. 1., s. 5). As s is very small for any sensible refraction, its powers can be neglected, and the above becomes by development

$$d\zeta = -\frac{a}{1-a}\cdot\frac{\sin z\,\frac{d\delta}{\delta_o}}{\left\{\cos^2 z - 2a\left(1-\frac{\delta}{\delta_o}\right) + 2s\sin^2 z\right\}^{\frac{1}{2}}} + s(\mathrm{A}), \qquad (10)$$

where A is simply an abbreviation for the first term on the right, and is for the present disregarded. By differentiation and substitution from the law of decrease of density, the above becomes

$$d\zeta = \frac{a}{1-a}\cdot\frac{\beta\sin z\,.\,e^{-\beta s}\,ds}{\{\cos^2 z - 2a\,(1-e^{-\beta s}) + 2s\sin^2 z)\}^{\frac{1}{2}}};$$

the same equation as obtained by Laplace. Assume, for the purpose of integrating this equation, a new variable, y connected with s by the equation :

$$s = y + \frac{\alpha(1-e^{-\beta s})}{\sin^2 z},$$

whence, by Lagrange's Theorem, expanding in powers of α

$$e^{-\beta s} = e^{-\beta y}\{1 - \text{terms involving } \alpha \text{ and its powers}\}. \qquad (11)$$

As α is supposed exceedingly small, and on substitution any terms involving α would give rise to α^2, these can be neglected, whence on substitution,

$$d\zeta = \frac{\alpha}{1-\alpha} \cdot \frac{\beta \sin z\, e^{-\beta y}\, dy}{\{\cos^2 z + 2y \sin^2 z\}}.$$

By assuming $y = \frac{t^2}{\beta} - \frac{1}{2}\cot^2 z$ this reduces itself directly to a well-known form:

$$\zeta = \frac{\alpha}{1-\alpha}\sqrt{2\beta}\, e^{\frac{1}{2}\beta\cot^2 z}\int e^{-t^2} dt.$$

For the horizontal refraction $\cot^2 z$ becomes zero, and the limits of the integral are from 0 to ∞, the latter being allowable, as whatever magnitude may be assigned to s above a small value it cannot affect the refraction; and under these conditions the integral becomes $\frac{1}{2}\Gamma\left(\frac{1}{2}\right)$ or $\frac{\sqrt{\pi}}{2}$ and

$$\zeta = \frac{\alpha}{1-\alpha}\sqrt{\frac{\beta\pi}{2}}. \qquad (12)$$

or Laplace's value neglecting powers of α. By retaining powers of α the above would have been the coefficient of a series, thus—

$$\{e^{-\alpha\beta} + 2^{\frac{1}{2}}\alpha.\beta\epsilon^{-2\alpha\beta} + \text{etc.}\}$$

By reducing the disregarded term in (10) in exactly the same manner, it becomes a similar transformation of the gamma function, and its value is $\frac{\alpha}{1-\alpha}\sqrt{\frac{\pi}{8\beta}}$ multiplied by the same series as the last, and thus it cannot be greater than $\frac{1}{2}$ of the above term (12). On the Moon β is about 33.

Restoring in (12) the value of β, it becomes

$$\zeta = \frac{\alpha}{1-\alpha}\sqrt{\left\{g_o\frac{\delta_o}{p_o}.a.\frac{\pi}{2}\right\}}. \qquad (13)$$

The above value of α depends on the density of the air, and may be written $\alpha' \delta_o'$, where α' is the refraction of air for the standard density on the Earth, and δ_o' represents the density of the air in terms of the standard density at the Earth's surface where the horizontal refraction occurs. Disregarding powers of α, and transposing,

$$\delta_o' = \frac{\zeta}{\alpha'} \sqrt{\left\{ \frac{2}{\pi} \cdot \frac{p_o}{g_o . \delta_o . a} \right\}} . \qquad (14)$$

Then, from the condition that the surface density shall be such that the density δ_o' at a height x above the surface shall not exceed a given value of ζ, by replacing the given value of δ_o' in equation (5),

$$\delta_o = \frac{\zeta}{\alpha' \sqrt{\left\{ \frac{2}{a\pi} \cdot \frac{p_o}{g_o \delta_o} \cdot \right\}}} e^{g_o \frac{\delta_o}{p_o} \cdot \frac{a}{a+x} \cdot x} \qquad (15)$$

and this is the maximum density at the surface for an atmosphere of uniform temperature whose horizontal refraction at the height x is equal to ζ.

The fundamental equation employed by Bessel, modifying slightly his notation, is,

$$\delta_o = \delta e^{\frac{m a_1^2}{l a^2} x},$$

obtainable from (5) by writing for $g_o \frac{\delta_o}{p_o}$ its value in terms of the gravity at the Earth's surface, by replacing $\frac{p_o}{\delta_o}$ by l and g_o by $m \frac{a_1^2}{a^2}$, where m is the mass of the Moon, and a the radius of the Earth, the constant quantity $\frac{p_o}{g_o \delta_o}$ being the height of the column of air of density δ_o necessary under action of gravity g_o to produce a pressure equal to p_o, whilst Bessel's l is the same when the force of gravity is supposed equal to that at the surface of the earth; and introducing the approximation employed by Bessel that $\frac{a\,x}{a+x}$ can be replaced by x. The laws of decrease of density found are therefore alike, so therefore should be the results. The expression for the refraction Bessel derives from the *Méc. Cel.* p. 252 :

$$\zeta = \alpha \sqrt{\left\{ \frac{a}{l} \cdot \frac{\pi}{2} \right\}} .$$

which is Laplace's result, obtained at (12) after writing $1 - a$ as unity and replacing β by $\frac{a}{l}$, which is Laplace's expression; and then in the same way as (15) is obtained, Bessel finds

$$\delta_o = \frac{\zeta}{a'}\sqrt{\left\{\frac{2}{\pi}\cdot\frac{l}{a}\right\}}\, e^{\frac{m a_1^2}{l a^2}x}.$$

Here Bessel assumes that the value of l in Laplace's equation is identical with his own, but Laplace states $l = \frac{p_o}{g_o \delta_o}$; and to convert it into Bessel's constant l or $\frac{p_o}{\delta_o}$, it must be multiplied by g_o or $\frac{m a_1^2}{a^2}$, which though taken into account in obtaining the law of decrease of density Bessel has neglected to do in applying the equation for the refraction, and his result therefore is imperfect. Replacing Laplace's l, by its value in terms of Bessel's or $\frac{l a^2}{m a_1^2}$, the equation above becomes

$$\delta_o = \frac{\zeta}{a'}\sqrt{\left\{\frac{2}{a\pi}\cdot\frac{l a^2}{m a_1^2}\right\}}\, e^{\frac{m a_1^2}{l a^2}x},$$

which is identical with that obtained at (15) and converted into it by restoring to x its proper value $\frac{a\,x}{a+x}$ and replacing $\frac{l a^2}{m a^2}$ by its value $\frac{p_o}{g_o \delta_o}$, or Bessel's result must be increased by $\sqrt{\frac{1}{g_o}}$ or $\frac{a}{a_1\sqrt{m}}$.

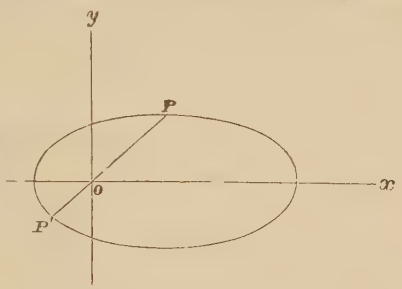
Bessel, employing Lindenau's value for the mass, and Burckhardt's for the diameter of the Moon, found the maximum surface-density for a lunar atmosphere exerting a horizontal refraction t the height of 5 miles above the surface of only one second of arc to be $\frac{1}{963}$rd of that of the Earth. Employing, however, the corrected expression, this is raised at once to $\frac{1}{377}$; but as already mentioned, the condition of a uniform temperature is not (as Bessel considered it to be) the most favourable for ensuring a moderately quick fall of temperature, and the surface density is raised at once to $\frac{1}{240}$th of that for the Earth, or more than four times as great as supposed possible by Bessel.

As in the paper by Bessel, the results of the foregoing are applicable to any planet as well as the Moon; but for a considerable

surface-density, if accuracy is required, it will be necessary to include the square of a, and for small planets the term s (A) in (10); in the larger planets this will be unnecessary, but the cube of a must be retained.

On a Theorem in Elliptic Motion. By Prof. Cayley.

Let a body move through apocentre between two opposite points of its orbit, say from the point P, excentric anomaly u, to the point P′, excentric anomaly u', where u, u' are each positive, $u < \pi$, $u' > \pi$. Taking the origin at the focus, and the axis of x in the direction through apocentre, then—



Coordinates of P are $x = a(-\cos u + e)$, $y = a\sqrt{1-e^2}\sin u$.

" P′ " $x = a(-\cos u' + e)$, $y = a\sqrt{1-e^2}\sin u'$;

whence, expressing that the points P, P′ are in a line with the focus,

$$\sin u'(-\cos u + e) - \sin u(-\cos u' + e) = 0,$$

that is

$$\sin(u'-u) = e(\sin u' - \sin u),$$

which is negative, viz. $u' - u$ is $> \pi$.

The time of passage from P to P′ is

$$\begin{aligned} nt &= (u' - e\sin u') - (u - e\sin u), \\ &= u' - u - e(\sin u' - \sin u), \\ &= u' - u - \sin(u'-u), \end{aligned}$$

which, $u' - u$ being greater than π, and $-\sin(u' - u)$ positive, is greater than π; viz. the time of passage is greater than one-half the periodic time. Of course if P, P′ are at pericentre and apocentre, the time of passage is equal one-half the periodic time.

The time of passage from P′ to P through pericentre is—

$$nT = 2\pi - (u'-u) + \sin(u'-u),$$

which is

$$= 2\pi - (u'-u) - \sin\{2\pi - (u'-u)\},$$

where $2\pi - (u' - u)$, $= \alpha$ suppose, is an angle $< \pi$. Writing, then

$$nt = \alpha - \sin \alpha,$$

and comparing with the known expression for the time in the case of a body falling directly towards the centre of force, we see that the time of passage from P′ to P through pericentre, is equal to the time of falling directly towards the same centre of force, from rest at the distance $2a$ to the distance $a(1 + \cos \alpha)$, where, as above $\alpha = 2\pi - (u' - u)$, $u' - u$ being the difference of the excentric anomalies at the two opposite points P, P′. If $\alpha = \pi$, the times of passage are each $= \frac{\pi}{n}$, that is, one half the periodic time.

The foregoing equation $\sin(u' - u) = e(\sin u' - \sin u)$ gives obviously

$$\cos \tfrac{1}{2}(u' - u) = e \cos \tfrac{1}{2}(u' + u);$$

that is

$$1 + \tan \tfrac{1}{2} u \tan \tfrac{1}{2} u' = e\left(1 - \tan \tfrac{1}{2} u \tan \tfrac{1}{2} u'\right),$$

or

$$-\tan \tfrac{1}{2} u \tan \tfrac{1}{2} u' = \frac{1-e}{1+e};$$

(in the figure $\tan \frac{1}{2} u$ is positive, $\tan \frac{1}{2} u'$ negative); and we thence obtain further

$$\sin \tfrac{1}{2}(u' - u) = \cos \tfrac{1}{2} u' \cos \tfrac{1}{2} u \left(\tan \tfrac{1}{2} u' - \tan \tfrac{1}{2} u\right),$$

$$\sin \tfrac{1}{2}(u' + u) = \cos \tfrac{1}{2} u' \cos \tfrac{1}{2} u \left(\tan \tfrac{1}{2} u' + \tan \tfrac{1}{2} u\right),$$

$$\cos \tfrac{1}{2}(u' - u) = \cos \tfrac{1}{2} u' \cos \tfrac{1}{2} u \cdot \frac{2e}{1+e},$$

$$\cos \tfrac{1}{2}(u' - u) = \cos \tfrac{1}{2} u' \cos \tfrac{1}{2} u \cdot \frac{2}{1+e};$$

and thence also

$$\cos u + \cos u' = 2 \cos \tfrac{1}{2}(u' + u) \cos \tfrac{1}{2}(u' - u),$$

$$= \cos^2 \tfrac{1}{2} u' \cos^2 \tfrac{1}{2} u \cdot \frac{8e}{(1+e)^2}.$$

But we have

$$1 + \cos(u' - u) = 2 \cos^2 \tfrac{1}{2}(u' - u) = \cos^2 \tfrac{1}{2} u' \cos^2 \tfrac{1}{2} u \cdot \frac{8e^2}{(1+e)^2},$$

or comparing with the last equation,

$$1 + \cos(u' - u) = e(\cos u + \cos u'),$$

or, what is the same thing,

$$1 - \cos(u' - u) = (1 - e\cos u') + (1 - e\cos u);$$

and in like manner,

$$1 + \cos(u' + u) = 2\cos^2\frac{1}{2}(u' + u) = \cos^2\frac{1}{2}u'.\cos^2\frac{1}{2}u\frac{8}{(1+e)^2};$$

or comparing with the same equation,

$$1 + \cos(u' + u) = \frac{1}{e}(\cos u + \cos u'):$$

which are formulæ corresponding with the original equation,

$$\sin(u' - u) = e(\sin u' - \sin u).$$

Note on the supposed Trapezoid Shape of the Sun's Corona.
By S. M. Drach, Esq.

From Mr. Proctor's "Sun as Ruler of Planetary System," coloured plate, p. 343, of Mr. Gilman's View of Eclipse of Aug. 7, 1869, I suggest that the contour *may* be the evolute of an ellipse, or the reflexion-caustic, when the reflector is an ellipse, ◇, and the radiant point in the centre (Coddington's *Optics*, p. 24, 25, fig. 22, 23, &c.). Long before Vassenius, in 1733, viz. at the great eclipse of May 1, 1706, the rose-coloured prominence was noticed by Capt. Stannyan, at Berne (first Flamstead says), and the corona, by M. Duillier, at Geneva, and also at Leipzic, *vide* Flamstead's Report, *Phil. Trans.*, No. 306, p. 2237; Hutton's *Abridgment*, v. 295; Hutton's *Mathematics of Ladies' Diaries*, i. 35–39, &c.

1875, *April* 5.

On the Star 61 *Geminorum.* By the Rev. T. W. Webb.

The star known as 61 *Geminorum* was entered in the Bedford Catalogue as double, $7\frac{1}{2}$ and 9 magnitudes, with a position-angle of 110°, and a distance of 60″, the colour being recorded as deep yellow, and yellowish.

I examined it with a $3\frac{7}{10}$-inch achromatic, March 23, 1852, and entered it "white, single." There could be little doubt as to its identification, from the neighbourhood of the double star H III. 48. February 14, 1855, having taken pains as to its identification, I again found it, with the same instrument, single and white. I noticed, however, on this latter occasion an exceedingly minute star, not above 11 mag., which might agree as to distance, but with a very roughly estimated angle of 185° or 190°.

In 1861 and 1871 the *comes* was invisible to Mr. Knott, with all the advantage of a $7\frac{1}{3}$-inch Alvan Clark object-glass. I believe I have never looked for it since, but have been lately interested by finding that it has been recovered this spring by Herbert Sadler, Esq., of Honiton Rectory, the extreme acuteness of whose vision, in the use of a $6\frac{1}{2}$-inch silvered mirror by Calver, is attested by his recognition of several most delicate and difficult objects. He gives it only 12·5 mag. at about the right distance, but with an estimated angle of 160° to 165°.

It seems, therefore, probable that, unless we can suppose an error in the figure expressing Smyth's magnitude, we have here a variable star, which it would be desirable to examine closely, and with instruments capable of giving a definite value to the angle of position. The possible change of colour also in the principal star merits attention. My own $9\frac{1}{3}$-inch speculum being at the present time dismounted, with a view to the ultimate perfection of its figure, though previously very good, I am unable to contribute any information on the subject.

Note on the Theory of Precession and Nutation.
By Prof. Cayley.

We have in the dynamical theory of Precession and Nutation (see Bessel's *Fundamenta* (1818), p. 126),

$$C\frac{dp}{dt} + (B-A)\,qr = L\,S\,(x'y - xy')\,dm'\left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right),$$

$$A\frac{dq}{dt} + (C-B)\,rp = L\,S\,(y'z - yz')\,dm'\left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right),$$

$$B\frac{dr}{dt} + (A-C)\,pq = L\,S\,(z'x - zx')\,dm'\left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right),$$

where L is the mass of the Sun or Moon, x, y, z the coordinates of its centre referred to the centre of the Earth as origin,

$$r = \sqrt{x^2 + y^2 + z^2},$$

the distance of its centre, and

$$\Delta = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2},$$

the distance of its centre from an element dm', coordinates (x', y', z') of the Earth's mass, the sum or integral S being extended to the whole mass of the Earth [I have written dm', r for Bessel's dm, r_1], we have

$$\Delta^2 = r^2 - 2(xx' + yy' + zz') + x'^2 + y'^2 + z'^2;$$

and thence

$$\frac{1}{\Delta^3} - \frac{1}{r^3} = \frac{3}{r^5}(xx' + yy' + zz')$$

$$- \frac{3}{2}\frac{1}{r^7}\{(x^2+y^2+z^2)(x'^2+y'^2+z'^2) - 5(xx'+yy'+zz')^2\}$$

$$+ \text{etc.}$$

The principal term is the first one,

$$\frac{3}{r^5}(xx' + yy' + zz');$$

but Bessel takes account also of the second term,

$$-\frac{3}{2}\frac{1}{r^7}\{(x^2+y^2+z^2)(x'^2+y'^2+z'^2) - 5(xx'+yy'+zz')^2\},$$

viz. considering the Earth as a solid of revolution (as to density as well as exterior form), he obtains in regard to it the following terms of $\sin\omega \dfrac{d\psi}{dt}$ and $\dfrac{d\omega}{dt}$ respectively ;

$$\frac{3L}{4r^4}\frac{1}{Cn} \cdot 2(C-A)K(5\sin^2\delta - 1)\cos\delta\sin\alpha,$$

$$-\frac{3L}{4r^4}\frac{1}{Cn} \cdot 2(C-A)K(5\sin^2\delta - 1)\cos\delta\cos\alpha,$$

where

$$2(C-A)K = S(3\mu - 5\mu^3)\, 2\pi\rho\, R^5\, dR\, d\mu,$$

K being in fact a numerical quantity, relating to the Earth only, and the value of which is by pendulum observations ultimately found to be $= 0{\cdot}13603$.

Writing, for shortness,

$$(x^2+y^2+z^2)(x'^2+y'^2+z'^2) - 5(xx'+yy'+zz')^2 = \Omega,$$

then the foregoing terms of $\sin\omega \dfrac{d\psi}{dt}$ and $\dfrac{d\omega}{dt}$ depend as regards their form on the theorem, that for any solid of revolution (about the axis of z) we have

$$S(x'y-xy')\,\Omega\, dm', \quad S(y'z-yz')\,\Omega\, dm', \quad S(z'x-zx')\,\Omega\, dm'$$

$$= 0,$$

$$\frac{1}{2}\, y\,(x^2+y^2+z^2-5z^2)\, S\left[3(x'^2+y'^2+z'^2)-5z'^2\right] z'\, dm',$$

$$-\frac{1}{2}\, x\,(x^2+y^2+z^2-5z^2)\, S\left[3(x'^2+y'^2+z'^2)-5z'^2\right] z'\, dm',$$

respectively: viz. writing $x'^2+y'^2+z'^2 = R^2$, and $z' = R\mu$, also $x^2+y^2+z^2 = r^2$ and $x = r\cos\delta\cos\alpha$, $y = r\cos\delta\sin\alpha$, $z = r\sin\delta$, the values would be

$$0,$$

$$\frac{1}{2}\, r^3 \cos\delta\sin\alpha\,(1-5\sin^2\delta)\, S(3-5\mu^2)\,\mu\, R^3\, dm',$$

$$-\frac{1}{2}\, r^3 \cos\delta\sin\alpha\,(1-5\sin^2\delta)\, S(3-5\mu^2)\,\mu\, R^3\, dm',$$

which are of the form in question.

The verification is easy: the solid being one of revolution about the axis of z, any integral such as $Sx'z'^2 dm'$ or $Sx'y'z'\, dm'$ which contains an odd power of x' or of y' is $= 0$; while such integrals as $Sx'^2 z'\, dm'$, $Sy'^2 z'\, dm'$ are equal to each other, or, what is the same thing, each $= \frac{1}{2} S\,(x'^2+y'^2)\, z'\, dm'$. That we have $S(x'y-xy')\,\Omega\, dm' = 0$ is at once seen to be true; considering the next integral $S(y'z-yz')\,\Omega\, dm'$, the terms of $(y'z-zy')\,\Omega$ which lead to non-evanescent integrals are

$$-y z' . (x^2+y^2+z^2)(x'^2+y'^2+z'^2),$$
$$-5\, y'z . 2\, y z\; y' z'.$$
$$+5\, y z' . (x^2 x'^2 + y^2 y'^2 + z^2 z'^2);$$

giving in the integral the several terms

$$-y (x^2+y^2+z^2) \mathrm{S} (x'^2+y'^2+z'^2) z'\, dm',$$
$$-10 y z^2 . \frac{1}{2} \mathrm{S} (x'^2+y'^2+z'^2-z'^2)\, dm',$$
$$+5\, y (x^2+y^2+z^2-z^2) . \frac{1}{2} \mathrm{S} (x'^2+y'^2+z'^2-z'^2)\, z'\, dm'$$
$$+y z^2 \mathrm{S} z'^3\, dm,$$

viz., collecting, the value is

$$\left(-1+\frac{5}{2}=\right) \quad \frac{3}{2} (x^2+y^2+z^2) y \mathrm{S} (x'^2+y'^2+z'^2) z'\, dm',$$
$$\left(-\frac{5}{2}=\right) -\frac{5}{2} (x^2+y^2+z^2) y \Sigma z'^3\, dm',$$
$$\left(-\frac{5}{2}-5=\right) -\frac{15}{2} y z^2 \mathrm{S} (x'^2+y'^2+z'^2) z'\, dm',$$
$$\left(+\frac{5}{2}+5+5=\right) +\frac{25}{2} y z^2 \mathrm{S} z'^3\, dm';$$

which is

$$= \frac{1}{2} y (x^2+y^2+z^2-5z^2) \mathrm{S} [3(x'^2+y'^2+z'^2)-5z'^2] z'\, dm';$$

and similarly the last term is

$$= -\frac{1}{2} x (x^2+y^2+z^2-5z^2) \mathrm{S} [3(x'^2+y'^2+z'^2)-5z'^2] z'\, dm',$$

which completes the proof.

Mr. R. A. Proctor writes:

I should be glad if students of astronomy would co-operate with me as follows:—In the historical portion of a work on the *Principles of Astronomy*, I am anxious to render strict justice in matters of priority, independent discovery, and so forth. Experience has taught me that this is not always easy; and I have, at times, been disquieted by communications pointing out errors and omissions on such points in former works of mine. But it might be possible to secure historical justice, if students of astronomy, especially any who consider that just claims have been overlooked, would call my attention to evidence of priority in matters of astronomical research, stating date and place of publication, or, preferably, sending copies of papers necessary and sufficient to establish priority. European astronomers should address to me at "2 North Road, Clapham Park, S.W." (till next September, when I leave England for the United States). Americans should address—"Care of the Literary Bureau, Cooper Institute, New York," (between October 1875 and July 1876).

Though I may probably be seldom able to reply to letters, every communication will be carefully considered.

Erratum.

Monthly Notices for 1875, March 12.

Page 294, line 10—the sentence "Modified equivalent of $\frac{a}{r}$ and $\frac{a}{r} - 1$" ought to have been included in one line. The words do not apply to the quantities which follow.